A CHANCE-CONSTRAINED OPTIMIZATION APPROACH FOR AIR TRAFFIC FLOW MANAGEMENT UNDER CAPACITY UNCERTAINTY

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Abstract

Airport capacity limitations remain a major problem for air traffic management. However, in the presence of capacity uncertainties, the ATFM operations may be impractical or ineffective when adopting the deterministic models since the latter assumes that airport capacity is known. In this paper, we propose a new approach based on Chance-Constrained Optimization Program (C-COP) taking into account airport capacity uncertainty to solve the Airport Network System Optimization (ANSO) problem. A scenario optimization method for airport capacity in the presence of uncertainties is suggested to approximately solve the C-COP with a predetermined probabilistic confidence. Then, a mixed-integer programming model is developed based on the obtained deterministic capacity to minimize the total flight delay. The experiment findings proved that our model is more effective and reliable in handling airport capacity uncertainty. Our results also highlight the ability of the C-COP to solve ANSO problem when airports’ capacities are uncertain.

Keywords: airport capacity, ATFM, uncertainty, mixed-integer programming, chance-constrained optimization, scenario approach

I. Introduction

Air transport plays a major role in the world’s global economic development. However, in the next few decades, air traffic may double in size if the current trends continue. Airport and airspace capacity limitations remain a major problem for air traffic management due to demand-capacity imbalances in many Chinese airports and sectors, especially during severe convective weather conditions. By the end of 2019, China's air traffic management system provided around 8.1 million air traffic control services for the major airports. However, this growing trend in air traffic and limitations in airport capacity has resulted in flight delays and airport congestion [1].

In the United States, San Antonio International Airport received 10.4 million passengers in 2019 with 19.03% of the flights being delayed or canceled and an average delay of 12.9 minutes per flight with an increase of 20.8% compared to the five previous years. Similarly, In China, according to 2019 statistics reported by the Civil Aviation Administration of China (CAAC), the annual number of passengers that Beijing Capital International Airport received were more than 100 million passengers, with an average delay of 21.7 minutes for each flight. The majority of these delays are created by imbalances between demand and capacity at the busiest airports during peak hours. Flight delays will not only be inconvenient for travelers, but they will also cost both passengers and airlines a significant amount of money [2].

An academic study published by researchers from three Chinese universities in 2018 showed that total flight delays in 2013 had indirectly cost the Chinese economy 350.7 billion yuan ($55 billion). These charges were calculated due to passengers' business and time losses as well as extra costs incurred by airports and airlines [3]. This situation might become more critical in the coming decade due to the rapid growth of air transport networks and capacity limitations.

To avoid delays, air traffic flow management (ATFM) has an important role to manage airport network capacity by appropriate demand-capacity balancing on both local and global levels and by ensuring the safety and the efficient usage of the national airspace. However, future ATFM operations will be hampered by the capacity limitations of airports and sectors. Airport capacity can be defined as the expected number of movements or the maximum number of operations (arrivals and
One of the challenges in improving ATFM performance through optimization is capacity uncertainty. For most airports, demand–capacity imbalance, airfield, runway configurations, air traffic control rules, and weather conditions are the most factors that affect the variation and loss of airport capacity. Adverse weather conditions such as strong winds, fog, precipitation, snow accumulation or ice on the runway can cut capacity significantly or shut down the airport altogether [5]. Therefore, airport capacity is generally quite complex to estimate accurately since airport capacity is composed of three interdependent types, namely airside capacity, terminal capacity, and landside capacity.

To provide a safe, orderly, expeditious, and effective air traffic operation in the airport, it appears necessary to develop an ATFM model taking into account capacity uncertainty to allocate airport resources, satisfy air traffic demand, and reduce traffic congestion. Several approaches and mathematical formulations for solving the ATFM problem have been proposed in the literature depending on the objectives and constraints of the problem. However, the majority of them simplify the real scenarios presuming that the capacity of the airport is known. These various approaches make ATFM operations less effective and impractical since the airport capacity is usually uncertain.

To this end, modeling uncertainty has gained significant attention. Over the past several years and is regarded as a critical subject in solving ATFM optimization problems [6]. Stochastic programming and Robust optimization are the most studied techniques to solve ATFM problems under uncertainty.

In stochastic programming, uncertainty is modeled by presuming that its probability distribution is known or can be precisely estimated based on its historical data. Although previous research considered this approach as useful and effective for solving ATFM problems, the probabilities are not frequently estimated accurately [7, 8].

Robust optimization on the other hand does not assume that probability distributions are known, it models the uncertainty in a deterministic way by assuming that the data lie within a closed set. The principal idea is to optimize the problem against the worst-case uncertainty scenario by solving a min-max optimization problem [9, 10].

Although both stochastic programming and robust optimization approaches have been successfully used to solve ATFM problems with uncertainty, they still suffer from overly conservative solutions.

In this paper, we propose a new approach based on Chance-Constrained Optimization Program (C-COP) that takes into account airport capacity uncertainty to solve the Airport Network System Optimization (ANSO) problem. The main goal is to minimize the flight delay costs and mitigate airport congestion taking into account airport capacity uncertainty.

First, we will illustrate a representative piecewise linear functions of airport capacity envelope approximation based on the Gilbo’s model [11]. To characterize the effect of operational uncertainty on capacity, a predefined probabilistic guarantee no less than \(1 - \epsilon\), where \(\epsilon \in (0,1)\) is assigned to the envelope. Then, we formulate the ANSO problem as a (C-COP). Due to the difficulty of solving this problem in terms of computational complexity, the C-COP is transformed into a standard convex optimization problem, a scenario approach is used to sample the uncertainty set and replace the random constraints with deterministic constraints. Finally, based on the obtained deterministic sector capacity, a mixed-integer programming (MIP) model is developed to minimize the total flight delay cost.

The rest of the paper is organized as follows: Section II presents the mathematical model of the ANSO problem. In Section III we explain the proposed approach to airport capacity estimation. Section IV presents the obtained results and assesses the effectiveness of the proposed approach. Section V provides a general conclusion and proposes some ideas for future work.

II. Mathematical Model

A. Problem Description

Consider a number of aircraft that are supposed to fly through specific airspace during a given time
interval, where some aircraft are on the ground at their original airports and some are airborne. However, at most airports, as demand over a certain period for flights approaches capacity, a large number of airports will be subject to the ‘high density rule’, which imposes hourly capacity limits and becomes increasingly congested. In other terms, the airport capacity is considered saturated when the maximum capacity is reached (no more parking places for the airplanes), which signifies that no more aircraft can be accepted to be held on the ground within a given period.

The majority of airport network research has been focused on samples of routes taken from airlines operating in a region [12]. In this study, we will focus on the airport capacity estimation and mitigates airport congestion propagation at the macroscopic level in the Chinese airport Network (CAN).

An air traffic network can be described as a directed graph \( G (N, E) \), where \( N \) denotes the network nodes (i.e., airports and sectors) and \( E \) denotes the set of air routes linking the nodes. The air traffic network has two types of nodes: airports and sectors. Assuming that the number of sectors and airports in the network is \( n \) and \( m \), respectively, then \( N = \{N_1, ..., N_n, N_{n+1}, ..., N_{n+m}\} \), where \( \{N_1, ..., N_n\} \) denotes the set of sectors, and \( \{N_{n+1}, ..., N_{n+m}\} \) denotes the set of airports. Figure 1 is an illustration of an air traffic network with spatial distribution of the sectors.

**Figure 1. Spatial Distribution of Sectors Network in Mainland of Chinese**
\[
P(f, i) = \begin{cases} 
\text{the departure airport, if } i = 1 \\
\text{the } (i - 1)^{\text{st}} \text{ sector in flight } f \text{'s path} \\
\text{if } 1 < i < N_f \\
\text{the arrival airport, if } i = N_f 
\end{cases}
\]

\[P_f = (P(f, i): 1 \leq i \leq N_f)\]

\[D_k(t) = \text{Departure capacity of airport } k \text{ at time } t\]

\[A_k(t) = \text{Arrival capacity of airport } k \text{ at time } t\]

\[S_j(t) = \text{Capacity of sector } j \text{ at time } t\]

\[d_f = \text{Scheduled departure time of flight } f\]

\[r_f = \text{Scheduled arrival time of flight } f\]

\[I_f = \text{Number of time units that flight } f \text{ must spend to cross sector } j\]

\[T_f^j = \left[ T_f^j, T_f^j \right] = \text{Set of feasible times for flight } f \text{ to arrive at sector } j \text{ (depart or arrive at airport } j)\]

\[T_f^j = \text{First feasible times for flight } f \text{ to arrive at sector } j \text{ (depart or arrive at airport } j)\]

\[T_f^j = \text{Last feasible times for flight } f \text{ to arrive at sector } j \text{ (depart or arrive at airport } j)\]

\[L_f^j = \text{Set of feasible flight level for flight } f \text{ in sector } j \text{ (airport } j)\]

\[\delta_f^j = \text{Maximum level variation for flight } f \text{ between sector } j \text{ (airport } j)\]

\[c_f^g = \text{Cost of holding flight } f \text{ on the ground for one unit of time}\]

\[c_f^a = \text{Cost of holding flight } f \text{ in the air for one unit of time}\]

3) Objective Function

The purpose of this optimization is to minimize total flight delay costs which include both ground holding and airborne delay costs. Bertsimas et al [14] has expressed the total number of time units that the flight \( f \) is kept on the ground as the difference between the actual departure time and the planned departure time, i.e.

\[g_f = \sum_{l \in T_f^p, k = P(f, i)} t(z_{k,i}^f - z_{k,i}^f(t - 1)) - d_f\]

while the total number of time units which flight \( f \) is kept in the air can be phrased as the actual arrival time minus the planned arrival time minus the period that a flight has been kept on the ground, i.e.

\[a_f = \sum_{l \in T_f^a, k = P(f, N_f)} t(z_{k,i}^f(t) - z_{k,i}^f(t - 1)) - r_f - g_f\]

With the variables \( g_f \) and \( a_f \), we can simply express the objective function as follows:

\[\min \sum_{f \in F} [c_f^g g_f + c_f^a a_f]\]

Finally, we can obtain the complete formulation of the objective function after substituting the expressions and rearranging the variables.

\[\sum_{f \in F} \left( c_f^g - c_f^a \right) \sum_{l \in T_f^p, k = P(f, i)} t(z_{k,i}^f - z_{k,i}^f(t - 1)) + c_f^a \sum_{l \in T_f^a, k = P(f, N_f)} t(z_{k,i}^f - z_{k,i}^f(t - 1)) + (c_f^g - c_f^a) d_f - c_f^a r_f\]

4) Constraints

\[\sum_{f \in F, j = f(i'), j+1 = f(i+1)} \sum_{l \in L_f^j} \left( z_{f,i}^f(t) - z_{f,i+1}^f(t) \right) \leq S_j(t) \forall j \in J, t \in T_f\]

\[\sum_{f \in F, j = f(i', j+1) = f(i+1)} z_{f,i}^f(t) \leq \sum_{f \in F, j = f(i')} z_{f,i+1}^f(t) \forall f \in F, 1 < i < N_f\]

\[\sum_{f \in F, j = f(i')} z_{f,i}^f(t) < 1 \forall f \in F, j \in P_f, t \in T_f\]

\[z_{f,i}^f(t) - z_{f,i}^f(t - 1) \geq 0 \forall f \in F, j \in P_f, l \in L_f^j, t \in T_f\]

\[z_{f,i}^f(t) - z_{f,i}^f(t - 1) = 0 \forall f \in F, j \in P_f, l \in L_f^j, t \in T_f\]

\[z_{f,i}^f(t) \in (0, 1) \forall f \in F, j \in P_f, l \in L_f^j, t \in T_f\]

Constraints (1) and (2) express the limitation of departure and arrival capacity of airport \( k \) at time \( t \). They ensure that the number of flights \( f \) that are allowed to take off from airport \( k \) at time \( t \) should not exceed the departure capacity of airport \( k \), and the number of flights that may arrive at airport \( k \) at time \( k \). constraint (3) assures that the sum of all flights \( f \) that arrive at sector \( j \) will not exceed the capacity of sector \( j \) at time \( t \). Constraints (4) indicate that if a flight \( f \) arrives at sector \( j \), it will also arrive
at its succeeding sectors. Constraints (5) indicate that each flight $f$ in a sector $j$ could not fly at more than one level at the same time. Constraints (6) ensure the temporal continuity of each flight. Constraints (7) guarantee that each flight has to arrive at the airport of destination at the last possible time $\bar{T}_k^f$. Constraint (8) represents the time window limitation for each flight as well as the sectors that a given flight is passing. Constraint (9) sets the model decision variables to be binary variables.

III. The Proposed Approach to Airport Capacity Estimation

As we have stated in Section II, the first and second constraints ensure that the number of flights $f$ that are allowed to take off from airport $k$ at time $t$ should not exceed the departure capacity of airport $k$, and the number of flights which may arrive at airport $k$ should not exceed the arrival capacity of airport $k$ at time $t$. However, due to the influence of various dynamic factors on airport capacity, e.g., fluctuations of meteorological conditions such as strong winds, fog, precipitation, snow accumulation, airport capacity at time $t$ is subject to uncertainties. Furthermore, uncertainties such as weather conditions could affect the robustness of the airport estimation result.

A. A Chance Constrained Optimization Program to Airport Capacity Estimation

To address the previously mentioned problem, first, we introduce a sequence of piecewise linear functions to describe the envelope model of the airport capacity. An airport capacity is defined as the sum of departures and arrivals flight counts that an airport can accommodate per hour, which is expressed as a function $y = f(x)$, where $x$ represents the number of arrival flights and $y$ represents the number of departure flights.

Suppose that a number of airplanes scheduled to fly through given airspace during a specific time interval are affected by airport capacity uncertainties which are indicated by $\delta \in \Delta$. Where $\Delta$ represents all possible uncertainty sets. Then, the airport capacity (number of departure and arrival flights) affected by uncertainty is written as $(x_\delta, y_\delta)$. A sequence of piecewise linear functions is introduced here (Figure 2) to describe the airport's capacity envelope.

The piecewise linear envelope functions of the airport capacity have the following formula [16][17]:

$$y = a_m + \beta_m x, \exists m \in [1, M], \text{ for } x \in [l_{m-1}, l_m] \quad (1\alpha)$$

The linear function of the arrival flight count is sampled uniformly with the interval $\Delta = \left[ \frac{l_{\text{max}}}{M} \right]$ from $[0, l_{\text{max}}]$ resulting in $M$ (quantity of all piecewise linear functions) intervals with the boundaries $l_m = m\Delta + \ell \in \mathbb{Z}$ where $m = 1, ..., M - 1$ and $l_0 = 0, l_M = l_{\text{max}}$. For example, $l_M = 1$ means that there are no departure flights, only arrival flights. The coefficients $(\alpha_m, \beta_m)$ represents the piecewise coefficients for the $m^{th}$ linear function of the arrivals flight count bounded by $[l_{m-1}, l_m]$. To guarantee the continuity of the envelope and the convexity of the region, the piecewise linear functions should satisfy respectively the two following constraints $(\alpha_m - \alpha_{m+1}) + l_m(\beta_m - \beta_{m+1}) = 0$ and $-\beta_m + \beta_{m+1} \leq 0$, where, $m = 1, ..., M - 1$.

To obtain a tight envelope, the optimization objective is transformed to minimize the area of the convex polygon region (the blue points part in Figure 2) based on the optimization variables $\{(a_m, \beta_m)\}_{m=1}^M$. The convex polygon region is enclosed by the capacity envelope function, the axes $(X, Y)$, and the line $L$, the area of polygon $h(a_m, \beta_m)$ is equal to the sum of areas of the M trapezoids shown as follows:

$$h(a_m, \beta_m) = \Delta l \sum_{m=1}^M \left[ a_m + \frac{l_{m-1} + l_m}{2} \beta_m \right] + (l_{\text{max}} - M\Delta l) \left( a_M + \frac{l_{M-1} + l_M}{2} \beta_M \right) \quad (2\alpha)$$
The capacity estimation problem is then formulated as a C-COP as follows:

\[
\min_{\alpha_m, \beta_m} h(\alpha_m, \beta_m)
\]

subject to:

\[
P[\delta \in \Delta: -\alpha_m - \beta_m x_\delta + y_\delta \leq 0 | \exists m \in [1, M], \text{for } x_\delta \in [l_{m-1}, l_m]] \geq 1 - \epsilon,
\]

\[-\beta_m + \beta_{m+1} \leq 0, m = 1, \ldots, M - 1,
\]

\[(\alpha_m - \alpha_{m+1}) + l_m(\beta_m - \beta_{m+1}) = 0, m = 1, \ldots, M - 1.\]

The optimization objective function (3a) is linear to the optimization variables. We assume that the probability distribution measure for the uncertainty set \(\Delta\) indicated by \(P\) is unknown to us. Here, a probabilistic constraint boundary is proposed so that all possible samples of departure flight based on arrival flight counts are fall inside the convex polygon area under the estimated capacity envelope except for a small violation probability no greater than \(\epsilon\) where \(\epsilon \in (0, 1)\), and the number of optimization variables is \(d = 2M - 1\).

### B. C-COP Solved Using a Scenario Optimization Approach

Since the probability distribution is unknown (non-Gaussian), the C-COPs such as (3a) are difficult to solve, and they may even be NP-hard in some particular cases [17, 18]. In reality, finding an approximate solution is the only way to solve these kinds of problems. Due to this, we resort to the scenario optimization technique to get an approximate solution to the C-COP mentioned above. The approach's basic idea is to replace the probabilistic constraint in (3a) with \(N\) set of deterministic constraints and maintaining the probabilistic guarantee by sampling the uncertainty set \(\delta \in \Delta\) with \(N\) scenarios \(\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(N)} \in \Delta\), keeping the same level of probability \(1 - \epsilon\). The formulation (3a) can be rewritten in the following way:

\[
\min_{\alpha_m, \beta_m} h(\alpha_m, \beta_m)
\]

subject to:

\[-\alpha_m - \beta_m x_{\delta^{(i)}} + y_{\delta^{(i)}} \leq 0, \exists m \in [1, M], \text{for } x_{\delta^{(i)}} \in [l_{m-1}, l_m], i = 1, \ldots, N,
\]

\[-\beta_m + \beta_{m+1} \leq 0, m = 1, \ldots, M - 1,
\]

\[(\alpha_m - \alpha_{m+1}) + l_m(\beta_m - \beta_{m+1}) = 0, m = 1, \ldots, M - 1.\]

The optimal solution \(\{(\alpha^*_m, \beta^*_m)\}_{m=1}^M\) with the probabilistic level \(1 - \epsilon\) to (4a) can be obtained by solving the standard linear program given in [19] as follows:

\[
\sum_{i=0}^{d-1} \binom{N}{i} \epsilon^i (1 - \epsilon)^{N-i} \leq \eta
\]

(5a)

where \(\eta \in (0, 1)\) is the confidence parameter, \(d\) is the dimension of optimization variables and \(\epsilon \in (0, 1)\) is the violation parameter, then the optimal solution \(\{(\alpha^*_m, \beta^*_m)\}_{m=1}^M\) satisfies that the

\[
P[\delta: -\alpha^*_m - \beta^*_m x_\delta + y_\delta \leq 0 | \exists m \in [1, M], \text{for } x_\delta \in [l_{m-1}, l_m]] \geq 1 - \epsilon
\]

with a confidence probability level no smaller than \(1 - \eta\). Note that, a reasonable \(\kappa\) is selected by considering the tradeoff between the computational
load and the performance of the solution. Particularly, if \( \kappa \to \epsilon \), the required number of scenarios \( N \to +\infty \).

C. A Posteriori Evaluation of The Capacity Envelope

A posteriori assessment method is used here to evaluate the robustness of the capacity envelope. Given the anticipated envelope, a test sample set is used to assess the actual violation probability \( \hat{\epsilon} \) by computing the samples outside the envelope. \( \hat{N}_{\text{Chernoff}} \) and \( \hat{N}_{\text{Bernoulli}} \) denote the number of test samples that are limited by Chernoff and Bernoulli bounds, respectively [21] [22]. This is considered as the principal bound for the Monte Carlo approach. Given an expected violation parameter \( \epsilon \in (0,1) \) and a confidence parameter \( \eta \in (0,1) \), the bound is shown as follows:

\[
\hat{N}_{\text{Che}} \geq \frac{1}{2\epsilon^2 \ln \left( \frac{2}{\eta} \right)}, \quad \hat{N}_{\text{Ber}} \geq \frac{1}{4\epsilon^2 \eta}
\]  

(8a)

IV. EXPERIMENTS

To assess the effectiveness of the Chance-Constrained Optimization approach for solving the ANSO problem. In this study, a Chinese airport network is established based on real data acquired from air traffic network services and an airport capacity estimation is created based on a normal distribution dataset. In this paper, a small-sized ATFM problem is developed which consists of 4 airports, 10 sectors, and 12 scheduled flights to evaluate the performance of C-COP and to perform the comparison. The flight plan is shown in Table 1. The ground handling and airborne holding costs of a flight \( f \) for one unit of time are given as \( c_f^g = 1 \) and \( c_f^a = 3 \), where each unit of time represents 5 minutes.

All the experiments for the implementation of the ANSO model are performed using a Python (version 3.6) interface with IBM ILOG CPLEX Optimization Studio (version 12.6.1), on a PC with Intel® Core™ i7-6700 processor, 3.40GHz, 16 GB RAM Microsoft Windows 7 OS.

A. Parameters Sets of The Proposed Approach

Based on the standard linear program given in (5a), we remark that the number of samples \( N \) is approximately proportional with the violation probability \( \epsilon \) (\( \epsilon \) goes to zero rapidly as \( N \) increases). Figure 3 shows how the violation probability varies with the number of required samples \( N \).

![Figure 3. Variation Of The Number Of Samples N With The Violation Parameter \( \epsilon \)](image)

As we can see, the required number of scenarios \( N \) decreases as \( \epsilon \) increases. This indicates that, for a properly high number of scenarios \( N \), the probability of obtaining a solution which is "poor" in terms of violation probability (i.e., having a violation greater than specified one) is extremely low.

As per the generalization theorem of the scenario approach, the number of \( N \) scenarios needed to ensure the desired level of robustness \( \epsilon \) grows as the number of optimization variables \( d \) increases, where \( d = 2M - 1 \) is determined by the number of linear segments \( M \).

<table>
<thead>
<tr>
<th>FLIGHTS</th>
<th>CROSSING TIMES</th>
<th>ORIGIN</th>
<th>DESTINATION</th>
<th>AIR ROUTE</th>
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<td>( f_1 )</td>
<td>[0,2,4,2,3,0]</td>
<td>( NKG )</td>
<td>( PVG )</td>
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<td>( f_2 )</td>
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<tr>
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<td>( PVG )</td>
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</tr>
<tr>
<td>( f_4 )</td>
<td>[0,2,4,2,3,0]</td>
<td>( NKG )</td>
<td>( PVG )</td>
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<tr>
<td>( f_5 )</td>
<td>[0,3,1,0]</td>
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<td>( WUX )</td>
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</table>
Figure 4 shows how the number of $N$ required scenarios changes with the number of optimization variables.

![Figure 4. Variation Of the Number of Samples $N$ With the Number of Optimization Variables $d$](image)

Simply put, the higher $M$ is, the more samples are required. Simultaneously, the more accurately the piecewise linear function approximates the real capacity envelope. The corresponding number of required samples variation for each $d$ and with the other parameters are shown in Table 2.

**Table 2. The Parameter Sets with Different Numbers of Optimization Variables**

<table>
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<th>$\epsilon$</th>
<th>$\eta$</th>
<th>$d$</th>
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<th>$K$</th>
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</tbody>
</table>

In this study, a trade-off compromise between the number of required samples and the piecewise linear function approximation with the parameters $M = 8$, $2M = 1 = 17$ is considered. The findings of the scenario approach are described in Table 3, for the two sets of violation probability $\epsilon = 0.05$ and $\epsilon = 0.102$ and for both cases with and without the number of constraints to be removed.

**Table 3. The Parameter Sets with Different Violation Probabilities**

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$\eta$</th>
<th>$d$</th>
<th>$N$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>$10^{-7}$</td>
<td>17</td>
<td>938</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.01</td>
<td>2191</td>
<td>19</td>
</tr>
<tr>
<td>0.102</td>
<td>$10^{-7}$</td>
<td>17</td>
<td>452</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.02</td>
<td>1030</td>
<td>18</td>
</tr>
</tbody>
</table>

To get a capacity envelope with 95% of probability guarantee, it is required to have $N = 2191$ samples, while the number of scenarios to be removed without decreasing the probability level is $K = 19$ scenarios ($\kappa = 0.01$). As soon as the probability level is lowered to 90%, the number of samples needed ($N = 1030$) has reduced by more than half, while the number of removed scenarios is $K = 18$ with $\kappa = 0.02$.

**B. Chernoff and Bernoulli Sample Size Comparison with Different Values Of $\epsilon$ and $\eta$**

A comparison of the sample size obtained with Chernoff and Bernoulli bounds is adopted here to assess the robustness of the capacity envelope. The problem is then to find an appropriate $N$ such that is satisfied for fixed violation probability $\epsilon \in (0,1)$ and confidence parameter $\eta \in (0,1)$. Table 4 gives a comparison of the sample size between Bernoulli and Chernoff bounds for various values of $\epsilon$ and $\eta$.

**Table 4. Comparison Of the Obtained Sample Size with Chernoff and Bernoulli Bounds for Various Values Of $\epsilon$ and $\eta$**

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$1 - \eta$</th>
<th>CHERNOFF</th>
<th>BERNOLLI</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.102</td>
<td>0.90</td>
<td>$1.50 \times 10^4$</td>
<td>$2.40 \times 10^4$</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>$1.84 \times 10^4$</td>
<td>$5.00 \times 10^4$</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>$2.64 \times 10^4$</td>
<td>$2.50 \times 10^5$</td>
</tr>
<tr>
<td>0.05</td>
<td>0.90</td>
<td>599</td>
<td>$1.00 \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>738</td>
<td>$2.00 \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>1060</td>
<td>$1.00 \times 10^4$</td>
</tr>
<tr>
<td>0.01</td>
<td>0.90</td>
<td>144</td>
<td>240</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>177</td>
<td>481</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>255</td>
<td>2403</td>
</tr>
</tbody>
</table>

We notice that the Chernoff bound improves largely upon the Bernoulli bound. This is because the Bernoulli sample size depends on $1/\eta$ while the Chernoff bound is a function of $\log (2/\eta)$. In addition, the dependence on $\epsilon$ is unchanged in both cases and it is reversely proportional to $\epsilon^2$. Hence, we conclude that the confidence parameter is “cheaper” than the violation probability.

**C. Estimated Airport Capacity Through the Proposed Approach**

Based on the proposed approach, the estimated airport capacity for 4 airports (NKG, WUX, SHA, PVG) is then given in Table 5 with the parameter $d = 17$, $\eta = 10^{-7}$.
To achieve an airport capacity with a 95% probabilistic guarantee. The proposed approach experiment is repeated 25 times to estimate the capacity of 4 airports using a normal distribution data set, where $N = 938$ samples are randomly selected for each execution and $\mu = 10, \sigma = 1.8$.

It should be mentioned that the airport capacity adopted here corresponds to the minimal boundary so as to mitigate the conservatism of the worst-case scenario of the uncertain parameters that appear in the dataset [23].

### D. Comparison Between the Deterministic Result and The Proposed Approach Result

To highlight the effectiveness of the scenario optimization approach for solving the ANSO problem. A small-sized ATFM problem was developed consisting of 4 airports, 10 sectors and 12 scheduled flights with departure and arrival times.

<table>
<thead>
<tr>
<th>Flight</th>
<th>(First, Last) SCHEDULED</th>
<th>Deterministic Solution Plans</th>
<th>Proposed Approach Solution Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Actual departure time</td>
<td>Actual arrival times</td>
</tr>
<tr>
<td>F1</td>
<td>(1,2)</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>F2</td>
<td>(2,3)</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>F3</td>
<td>(2,10)</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>F4</td>
<td>(2,10)</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>F5</td>
<td>(3,10)</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>F6</td>
<td>(4,10)</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>F7</td>
<td>(3,10)</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>F8</td>
<td>(3,10)</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>F9</td>
<td>(1,10)</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>F10</td>
<td>(1,10)</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>F11</td>
<td>(3,10)</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>F12</td>
<td>(2,10)</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total delay cost = 32.000000</td>
<td></td>
</tr>
</tbody>
</table>

All airport capacity = 9, at all times  
All airports capacity is listed in table 5
flights under the capacity scenario is considered in this experiment.

![Diagram](image)

**Figure 5. Demonstration of the Relationship Between Violation Probability and Total Delay Cost**

From Figure 5 we can see that the total delay cost gets higher when $\epsilon$ is smaller and it decreases when the violation probability is higher. This indicates that, due to the capacity uncertainty factors, the solver will adopt a more cautious strategy to increase airborne delays in order to cope with the possible scenarios to reduce the airport capacity. More crucially, based on the analysis results shown in Figure 5, the decision makers may use the ANSO model to optimize the flight plans for all planned flights by selecting the suitable violation probability based on their previous experience.

V. Conclusion and Future Research

In this paper, we proposed a scenario optimization technique to solve the ANSO problem in the presence of airport capacity uncertainty. We formulated the ANSO problem as a Chance-Constrained Optimization Program (C-COP) which we solved via the scenario optimization method under capacity uncertainties to approximate the optimal solution with a predetermined probabilistic confidence. To assess the effectiveness of our model, a small-sized air traffic network was developed consisting of 4 airports, 10 sectors and 12 scheduled flights.

To evaluate the effect of capacity uncertainties on total delay costs, we investigated the impact of the violation probability on the total delay cost. This can assist decision-makers in selecting an appropriate value of violation probability and accomplish an acceptable trade-off between optimization and robustness.

Future work will extend the results towards different kinds of uncertainty sources such as flight demand.

**References**


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